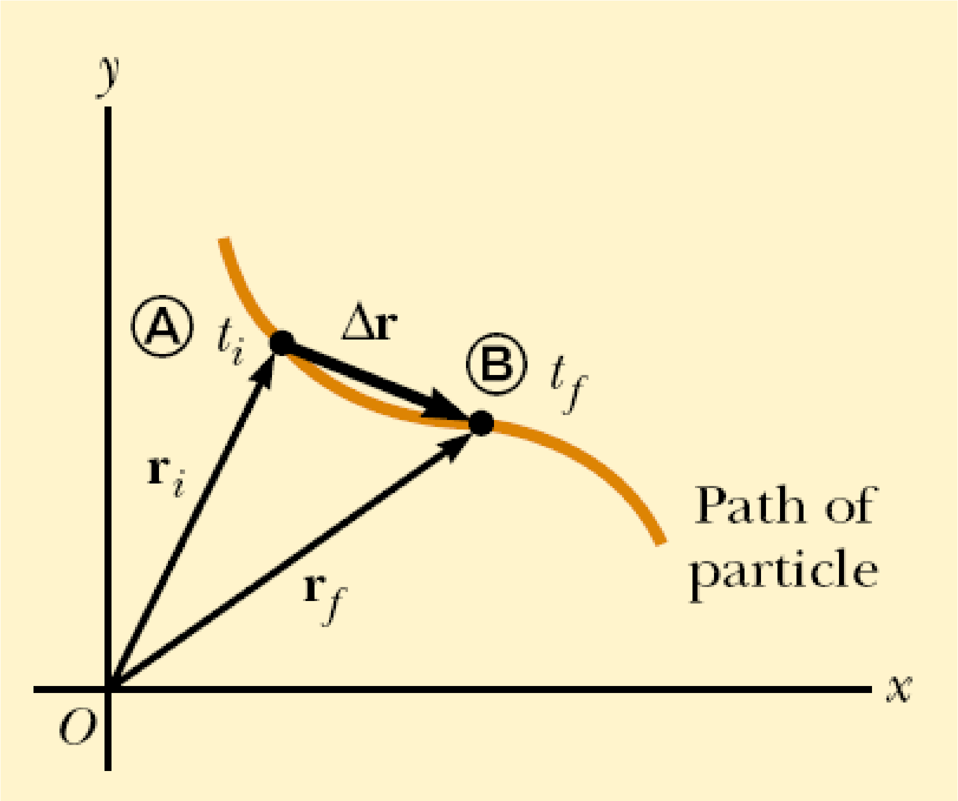
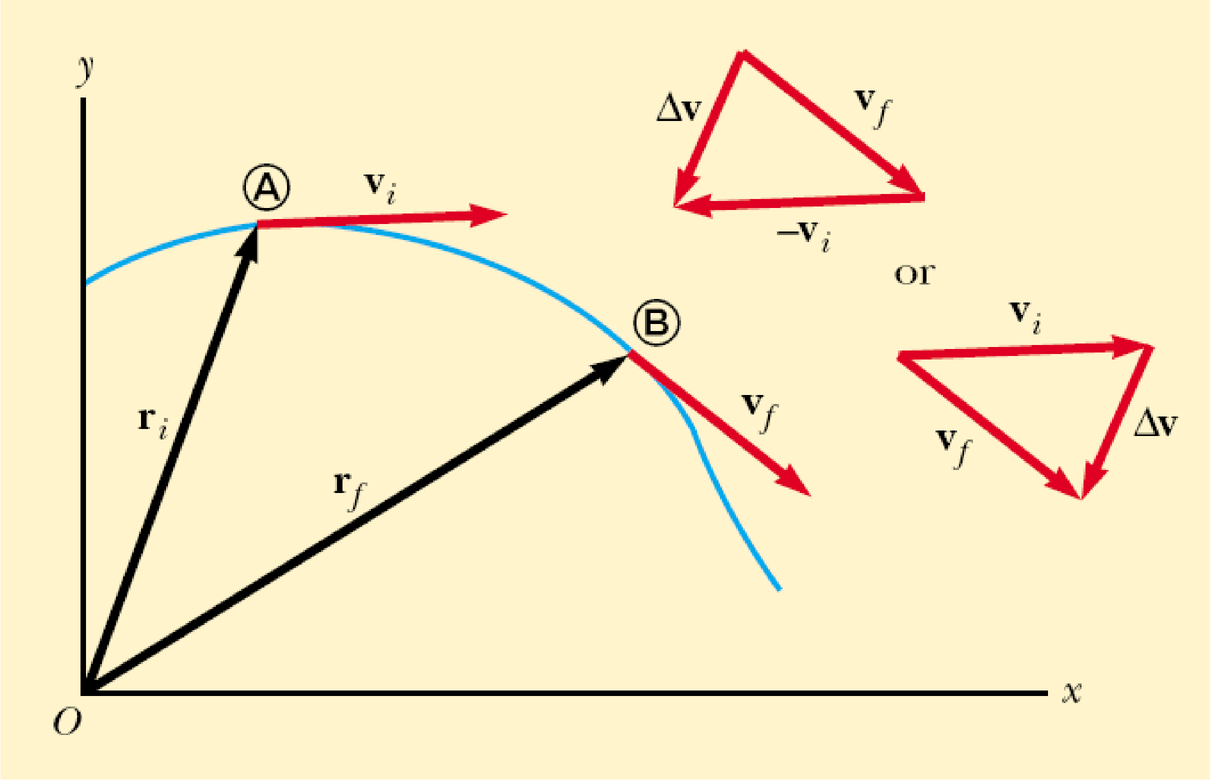
Zmiana położenia – zapis wektorowy



$$\vec{\mathbf{v}} \equiv \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta \mathbf{r}}}{\Delta t} = \frac{\overrightarrow{d\mathbf{r}}}{dt}$$

Prędkość chwilowa jako styczna do toru, def. przyspieszenia



$$\vec{\mathbf{v}} \equiv \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta \mathbf{r}}}{\Delta t} = \frac{\overrightarrow{d\mathbf{r}}}{dt}$$

$$\vec{\mathbf{a}} \equiv \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta \mathbf{v}}}{\Delta t} = \frac{\overrightarrow{d\mathbf{v}}}{dt}$$

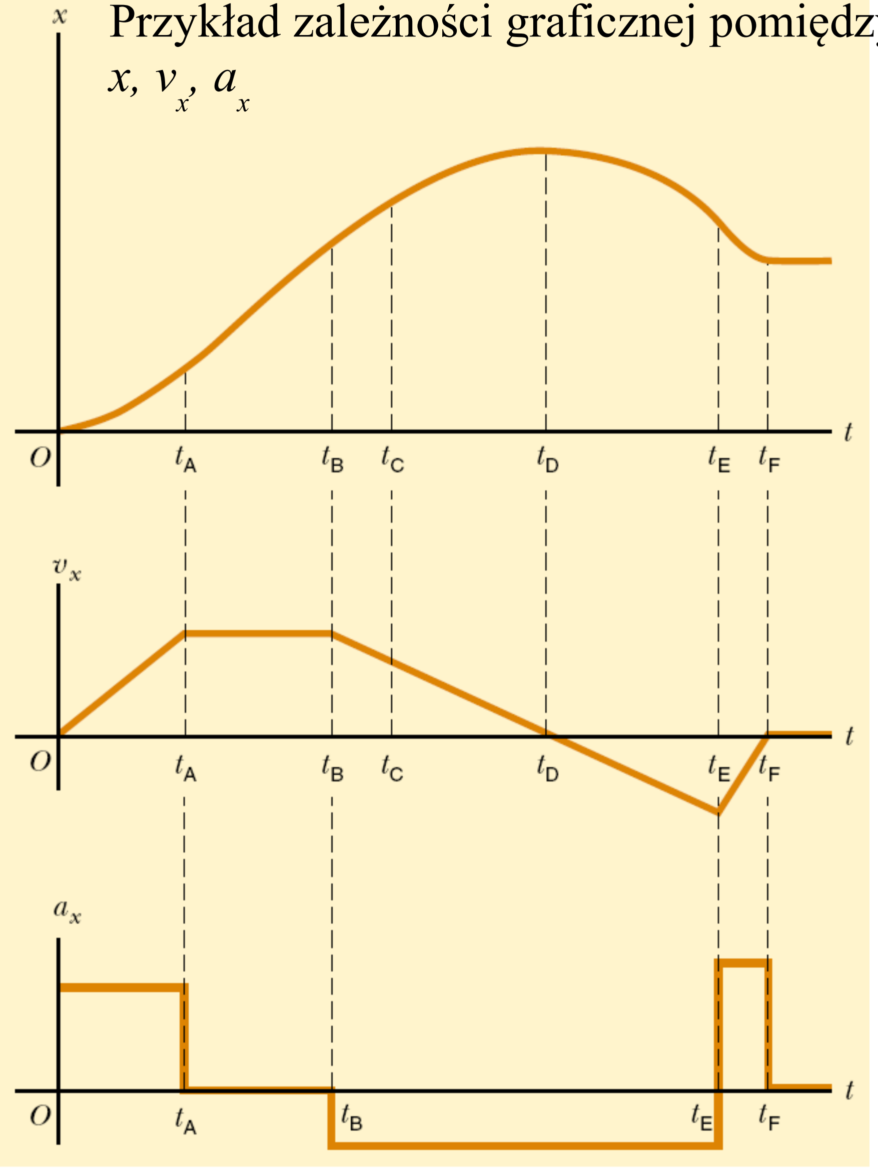
Ruch jednowymiarowy

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

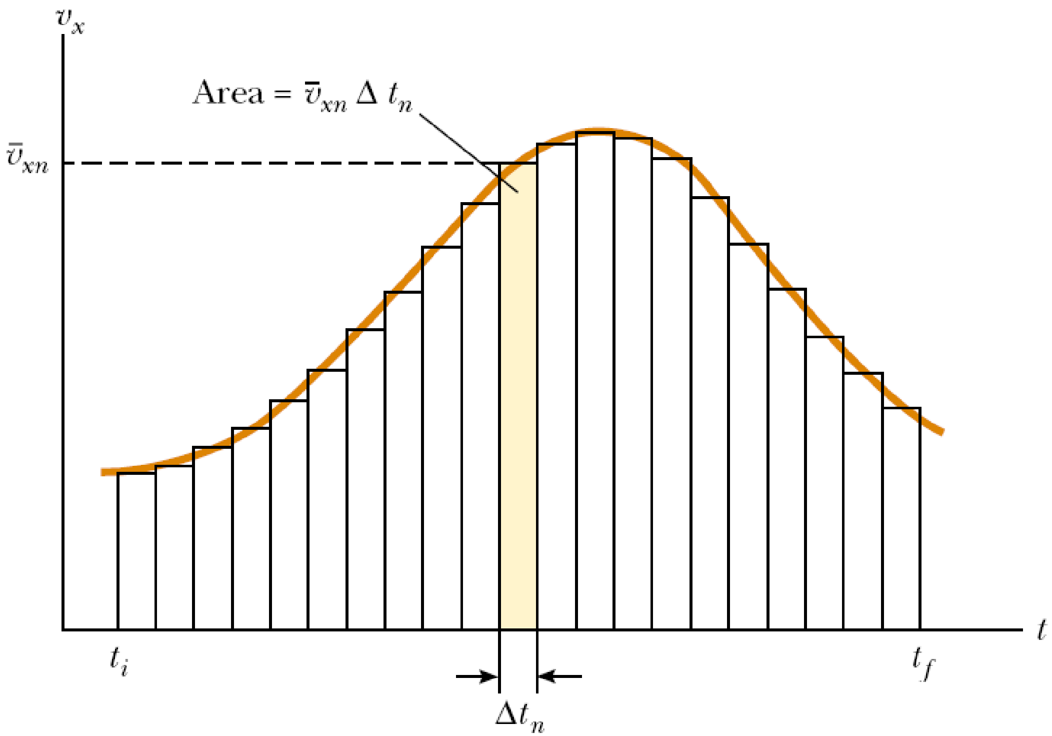
$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

Przykład zależności graficznej pomiędzy x, v_x, a_x



Jak policzyć drogę w ruchu o zmiennej prędkości



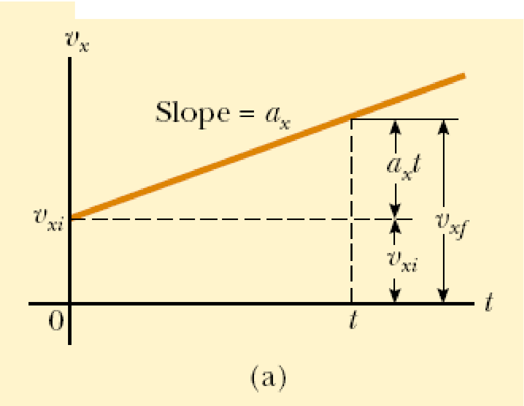
$$\Delta x = \sum_n \bar{v}_{xn}\,\Delta t_n$$

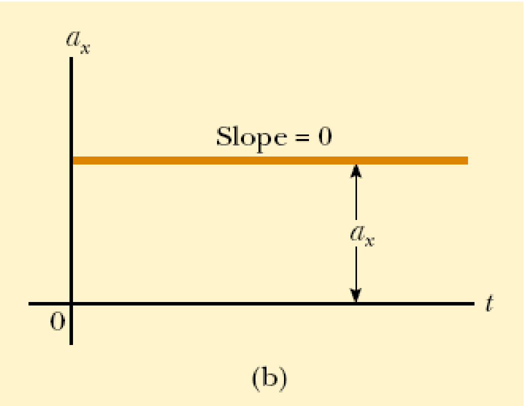
$$\Delta x = \lim_{\Delta t_n \to 0} \sum_n v_{xn}\,\Delta t_n$$

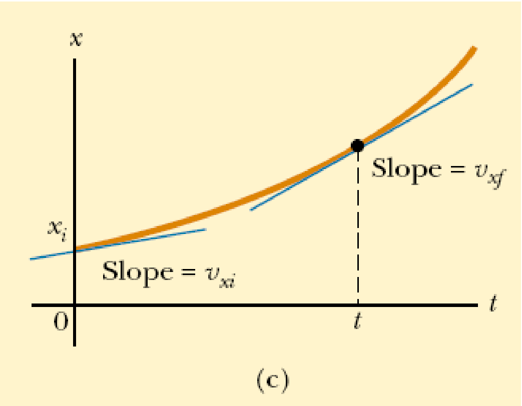
$$\lim_{\Delta t_n \to 0} \sum_n v_{xn}\Delta t_n = \int_{t_i}^{t_f} v_x(t)\;dt$$

Ruch jednowymiarowy, jednostajnie przyspieszony

$$a_x = \frac{v_{xf} - v_{xi}}{t}$$



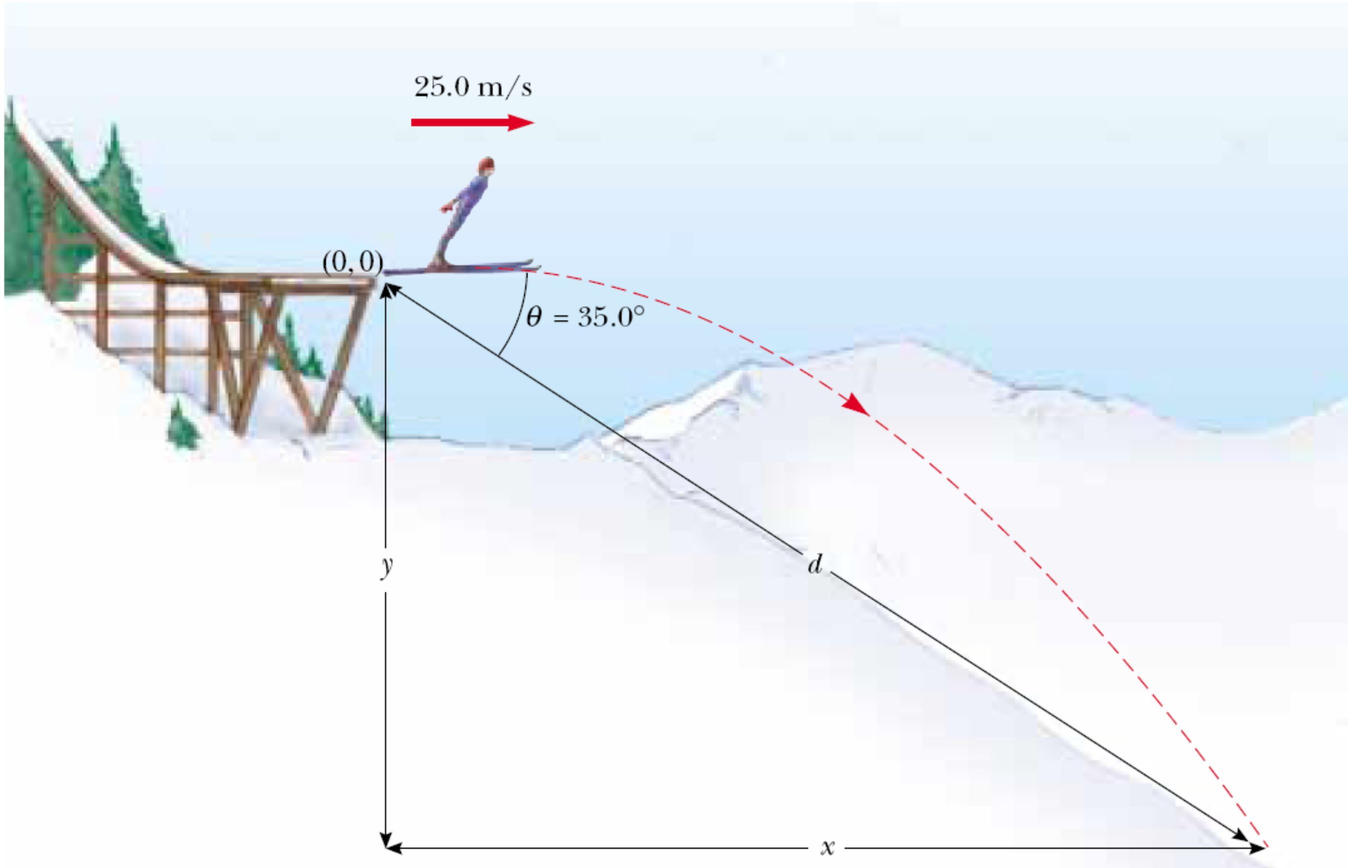




$$\bar{v}_x = \frac{v_{xi} + v_{xf}}{2}$$

$$v_{xf} = v_{xi} + a_x t$$
$$x_f - x_i = \tfrac{1}{2}(v_{xi} + v_{xf})t$$
$$x_f - x_i = v_{xi}t + \tfrac{1}{2}a_x t^2$$
$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

Przykład ruchu dwuwymiarowego



EXAMPLE 4.7 The End of the Ski Jump

A ski jumper leaves the ski track moving in the horizontal direction with a speed of 25.0 m/s, as shown in Figure 4.14. The landing incline below him falls off with a slope of 35.0°. Where does he land on the incline?

EXAMPLE 4.7 The End of the Ski Jump

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Solution It is reasonable to expect the skier to be airborne for less than 10 s, and so he will not go farther than 250 m horizontally. We should expect the value of d, the distance traveled along the incline, to be of the same order of magnitude. It is convenient to select the beginning of the jump as the origin ($x_i = 0, y_i = 0$). Because $v_{xi} = 25.0$ m/s and $v_{yi} = 0$, the x and y component forms of Equation 4.9a are

$$(1) \quad x_f = v_{xi}t = (25.0 \text{ m/s})t$$

$$(2) \quad y_f = \tfrac{1}{2}a_y t^2 = -\tfrac{1}{2}(9.80 \text{ m/s}^2)t^2$$

From the right triangle in Figure 4.14, we see that the jumper's x and y coordinates at the landing point are $x_f = d\cos 35.0°$ and $y_f = -d\sin 35.0°$. Substituting these relationships into (1) and (2), we obtain

$$(3) \quad d\cos 35.0° = (25.0 \text{ m/s})t$$

$$(4) \quad -d\sin 35.0° = -\tfrac{1}{2}(9.80 \text{ m/s}^2)t^2$$

Solving (3) for t and substituting the result into (4), we find that $d = 109$ m. Hence, the x and y coordinates of the point at which he lands are

$$x_f = d\cos 35.0° = (109 \text{ m})\cos 35.0° = \boxed{89.3 \text{ m}}$$

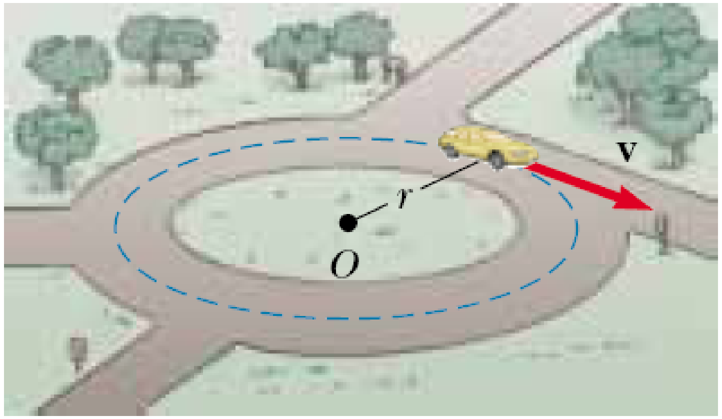
$$y_f = -d\sin 35.0° = -(109 \text{ m})\sin 35.0° = \boxed{-62.5 \text{ m}}$$

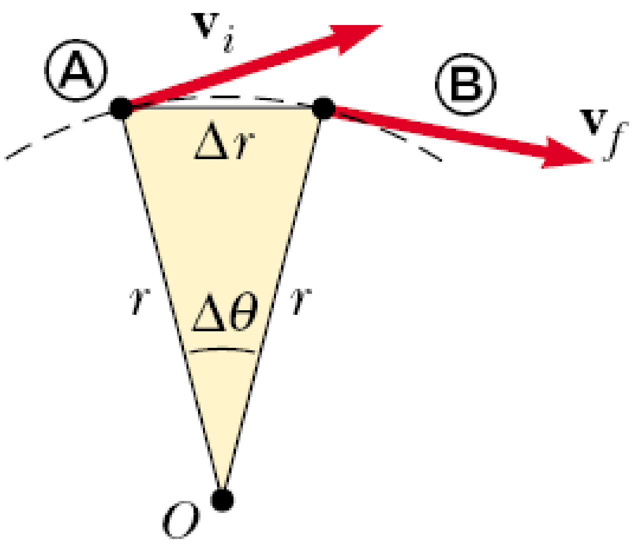
Exercise Determine how long the jumper is airborne and his vertical component of velocity just before he lands.

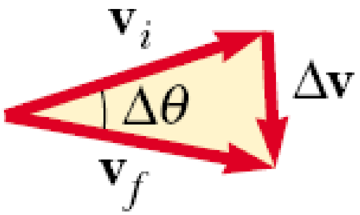
Answer 3.57 s; -35.0 m/s.

Jednostajny ruch po okręgu i przyspieszenie

- Stała co do wartości prędkość *v* (stała szybkość), ale zmienny kierunek prędkości

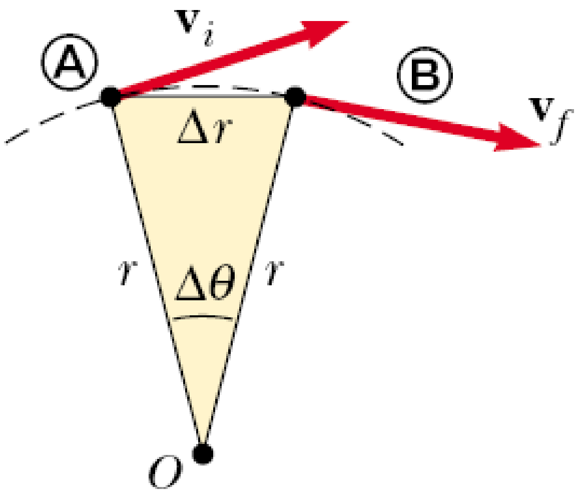


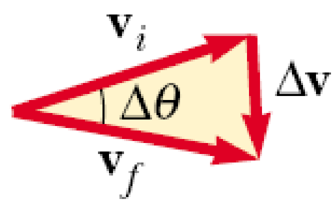




Jednostajny ruch po okręgu i przyspieszenie

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} = \frac{\vec{\Delta v}}{\Delta t}$$





$$\frac{\Delta v}{v} = \frac{\Delta r}{r}$$

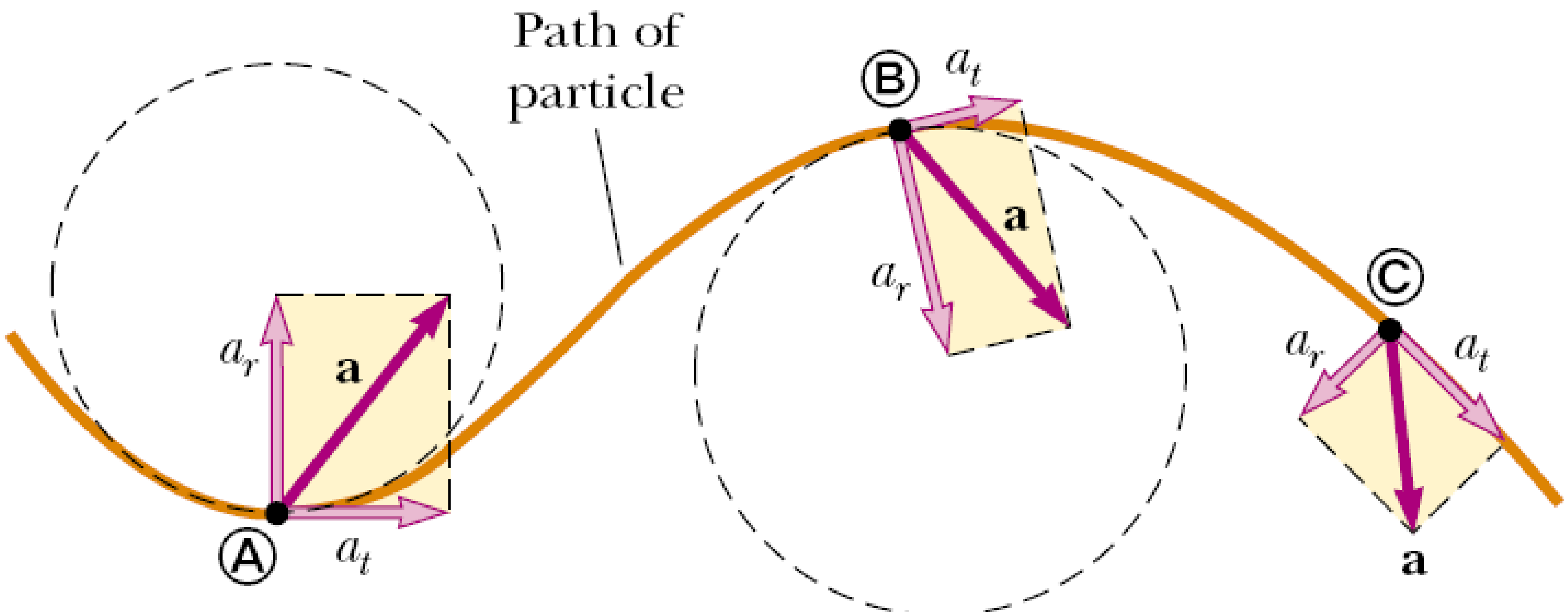
$$\vec{a} = \frac{v\,\vec{\Delta r}}{r\,\Delta t}$$

$$\Delta t \to 0, \quad \Delta r/\Delta t \to v.$$

$$|a_r| = \frac{v^2}{r}$$

Przyspieszenie „styczne” i „normalne”

- Jeśli obiekt porusza się ruchem zmiennym (zmienia się kierunek i wartość prędkości) to przyspieszenie możemy rozłożyć na dwie składowe: „styczną” do toru, „normalną” do toru.



$$\vec{\mathbf{a}} = \vec{\mathbf{a}}_r + \vec{\mathbf{a}}_t \qquad a_t = \frac{d|\vec{\mathbf{v}}|}{dt} \qquad a_r = \frac{v^2}{r}$$

Example 4.8 The Swinging Ball

A ball tied to the end of a string 0.50 m in length swings in a vertical circle under the influence of gravity, as shown in Figure 4.19. When the string makes an angle $\theta = 20°$ with the vertical, the ball has a speed of 1.5 m/s. (a) Find the magnitude of the radial component of acceleration at this instant.

Solution The diagram from the answer to Quick Quiz 4.4 (p. 109) applies to this situation, and so we have a good idea of how the acceleration vector varies during the motion. Figure 4.19 lets us take a closer look at the situation. The radial acceleration is given by Equation 4.18. With $v = 1.5$ m/s and $r = 0.50$ m, we find that

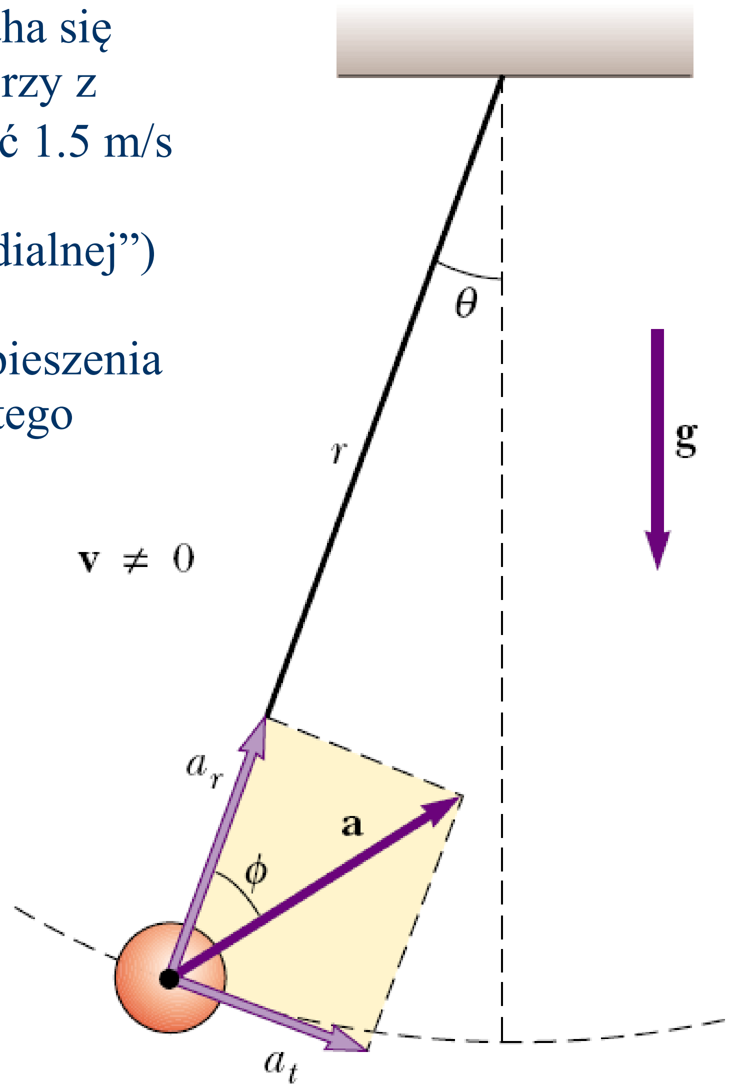
$$a_r = \frac{v^2}{r} = \frac{(1.5\ \text{m/s})^2}{0.50\ \text{m}} = 4.5\ \text{m/s}^2$$

(b) What is the magnitude of the tangential acceleration when $\theta = 20°$?

Przyspieszenie „styczne” i „normalne”: wahadło

Kulka wisi na lince o dł 0.5m i swobodnie waha się pod wpływem siły ciężkości. Kiedy linka tworzy z pionową osią kąt $\phi = 20^{o}$ piłka osiąga prędkość 1.5 m/s

a/ jaka jest wartość składowej normalnej („radialnej”) przyspieszenia
b/ jaka jest wartość składowej stycznej przyspieszenia
c /jaka jest wartość, zwrot i kierunek całkowitego przyspieszenia?



Solution When the ball is at an angle θ to the vertical, it has a tangential acceleration of magnitude $g \sin\theta$ (the component of $\mathbf{g}$ tangent to the circle). Therefore, at $\theta = 20°$,

$$a_t = g \sin 20° = 3.4 \text{ m/s}^2.$$

(c) Find the magnitude and direction of the total acceleration $\mathbf{a}$ at $\theta = 20°$.

Solution Because $\mathbf{a} = \mathbf{a}_r + \mathbf{a}_t$, the magnitude of $\mathbf{a}$ at $\theta = 20°$ is

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(4.5)^2 + (3.4)^2} \text{ m/s}^2 = 5.6 \text{ m/s}^2$$

If ϕ is the angle between $\mathbf{a}$ and the string, then

$$\phi = \tan^{-1}\frac{a_t}{a_r} = \tan^{-1}\left(\frac{3.4 \text{ m/s}^2}{4.5 \text{ m/s}^2}\right) = 37°$$

Note that $\mathbf{a}$, $\mathbf{a}_t$, and $\mathbf{a}_r$ all change in direction *and* magnitude as the ball swings through the circle. When the ball is at its lowest elevation ($\theta = 0$), $a_t = 0$ because there is no tangential component of $\mathbf{g}$ at this angle; also, a_r is a *maximum* because v is a maximum. If the ball has enough speed to reach its highest position ($\theta = 180°$), then a_t is again zero but a_r is a minimum because v is now a minimum. Finally, in the two horizontal positions ($\theta = 90°$ and $270°$), $|\mathbf{a}_t| = g$ and a_r has a value between its minimum and maximum values.

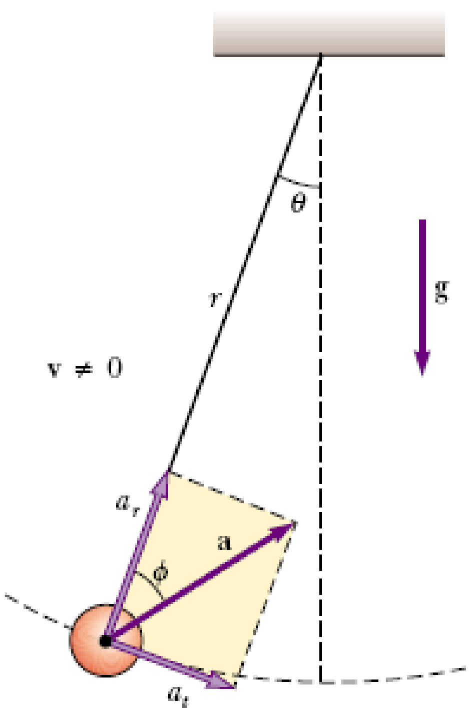


Figure 4.19 Motion of a ball suspended by a string of length r. The ball swings with nonuniform circular motion in a vertical plane, and its acceleration $\mathbf{a}$ has a radial component a_r and a tangential component a_t.

1-19. Szybkość prądu wody w rzece zmienia się wraz z szerokością rzeki według prawa $v = -4x^2 + 4x + 0,5$, gdzie $x = a/b$ (a — odległość od brzegu, b — szerokość rzeki). O jaki odcinek prąd wody w rzece zniesie łódkę przy przeprawie na drugi brzeg, jeżeli szybkość łódki względem wody równa się 2 m/s? Szybkość łódki ma kierunek prostopadły do brzegów rzeki. Szerokość rzeki 420 m.